INTRODUCTION

This paper considers the strategic currency hedging optimization problem for institutional investors with globally diversified portfolios. We argue the currency hedging decision should be made at the portfolio level on a currency-by-currency basis. It is suboptimal to develop hedging strategies by asset or to impose uniform or asset-specific hedging constraints at the portfolio level. Extant research on currency hedging usually confines the analysis to single asset classes rather than multi-asset portfolios (see, for example, Perold and Schulman (1988), Campbell, Serfaty-de Medeiros and Viceira (2010), Schmittmann (2010)). As a result, the recommended currency hedging strategies based on these analyses are often asset-specific.\(^1\) Some studies extend the analysis to the portfolio level but only consider uniform hedge ratios (Eun and Resnick (1988); Liu and Jacobsen (2014)). These approaches are also popular among practitioners globally.\(^2\) We quantify the potential efficiency loss of these common approaches relative to the recommended one. In our sample optimizations for Australian and Japanese investors, adopting currency-specific hedge ratios can reduce conditional value at risk (CVaR) by up to 1.5% and 3%, respectively, without reducing the expected return of the portfolio.

In addition, we propose an analytical framework based on marginal risk and return contributions to explain the optimal hedge ratios and the underlying drivers of the efficiency implications of the common practices. We use CVaR as our primary measure of risk in order to more accurately account for tail risk when investors don’t have quadratic utility functions and returns are not normally distributed. Agarwal and Naik (2004) argue that the left-tail is underestimated in the common mean-variance optimization and support the use of mean-CVaR optimization as an alternative.

To emphasize the importance of base currency for the currency hedging optimization problem, we consider three types of investors (Australian, U.S. and Japanese). The Australian dollar tends to positively correlate with equities and appreciate against the U.S. dollar and the Japanese yen during up markets. The yen is considered a “safe haven” currency for investors and during risk-off periods appreciates even against another safe haven currency, the U.S. dollar (Botman, de Carvalho Filho and Lam (2013)). As a result, in an effort to minimize risk, an Australian investor would leave most FX exposures unhedged, a Japanese investor would hedge all FX exposures, and a U.S. investor would hedge all FX exposures except for JPY.
OPTIMIZATIONS

For the purposes of this study, we assume exogenous asset allocations. For each base currency (AUD, JPY and USD), we construct a global 60/40 portfolio with 30% global equities, 30% domestic equities, 20% global bonds and 20% domestic bonds. We consider seven major currencies for hedging that, in total, represent more than 95% of the MSCI World Index and the Barclays Global Aggregate Index (as of 31 December 2015): USD, EUR, JPY, GBP, CAD, AUD and CHF. Appendix 1 shows the sample portfolios and capital market assumptions based on an internal PIMCO evaluation survey.

We define the optimal hedging strategy as the permissible hedging strategy that minimizes CVaR for a given level of expected return or maximizes expected return for a given level of CVaR. We estimate CVaR at the 95% confidence level nonparametrically with a block bootstrap methodology.

For each base currency, we perform mean-CVaR optimizations under three setups:

1. Uniform hedge ratio across all currencies
2. Asset-specific hedge ratios
3. Currency-specific hedge ratios

Appendix 2 shows a mathematical description of the optimization problem.

2.1 AUD Base Case

Exhibit 1 shows the efficient frontiers under the three types of hedging constraints: uniform, asset-specific and currency-specific for AUD investors.

The green line represents the efficient frontier given a uniform hedge ratio across all currencies. It is the most restrictive option and the least efficient among all three frontiers. The red line represents the frontier with asset-specific hedging constraints, and the blue line represents the frontier with currency-specific hedging constraints. At any given level of risk, the optimized portfolio return with currency-specific hedging constraints is higher than that with asset-specific hedging constraints. This is expected because the opportunity set for the former contains that for the latter. We can characterize the individual frontiers in the following way:

1. Uniform hedge ratio

The optimization is trivial in this case, as the frontier simply depends on the risk/return profile of the portfolio as a function of the uniform hedge ratio.

The return assumption for the single hedging asset in this optimization is 0.68%. The portfolio’s expected annual return will increase by about 7 basis points for every 10% increase in the uniform hedge ratio.

To understand how portfolio risk changes with the uniform hedge ratio, it is helpful to estimate the marginal contribution to CVaR (MCCVaR) of hedging as we change the hedge ratio from 0 to 1. A positive value of MCCVaR indicates a very small
increase in hedging would increase the portfolio risk, and vice versa. Exhibit 2 plots this measure conditional on the hedge ratio for the global 60/40 portfolio. It shows that a higher uniform hedge ratio almost always increases the portfolio CVaR except when the portfolio is completely unhedged or hedged up to 2.4%. This is intuitive since AUD is widely regarded as a risk-on currency relative to the other major developed market (DM) currencies. Generally speaking, exposure to other major DM currencies serves as a net risk diversifier rather than a contributor. However, FX exposure eventually becomes a risk contributor when the hedge ratio is lower than 2.4%. As a result, the efficient frontier (Exhibit 1) starts from the minimum-CVaR portfolio with a uniform hedge ratio of 2.4% and ends with the 100% hedged portfolio.

Exhibit 2: MCCVaR of uniform hedging ratio for the AUD global 60/40 portfolio

Exhibit 3 shows the marginal contribution to return (MCR) and marginal contribution to CVaR of the three different hedging assets for three levels of uniform hedging (2.4%, 50% and 100%).

The marginal contribution to return for each hedging asset is a constant equal to its expected return. As shown in Exhibit 20 in the appendix, the expected returns for the three hedging assets for the 60/40 portfolio, global equities and global bonds are 0.68%, 0.38% and 0.30%, respectively.

For a mean-CVaR optimization, the local attractiveness of a hedging asset for any given portfolio depends on its marginal contribution to return and CVaR. More specifically, for hedging assets in the first quadrant (MCR>0, MCCVaR>0) of Exhibit 3,
additional hedging increases both return and risk of the portfolio. We can measure the relative attractiveness of these hedging assets by their marginal rate of substitution (MRS) between expected return and risk, which is defined as the ratio of MCR to MCCVaR. Since both contributions are positive, the higher the MRS, the more attractive the hedging asset is for the optimization. If a hedging asset is in the second quadrant (MCR>0, MCCVaR<0), additional hedging can increase return while reducing risk, making it more attractive than any asset in the first quadrant.

Based on this intuition, we draw three lines in Exhibit 3. Each line starts from the origin and passes through the portfolio hedging asset at each of the three hedge ratios:

The 2.4% hedged portfolio is the minimum-CVaR portfolio under a uniform hedge ratio. If we now consider asset-specific hedging, the global equity hedging asset in the second quadrant (MCR>0, MCCVaR<0) is more attractive than the portfolio hedging asset (MCR>0, MCCVaR=0) because it marginally reduces risk as well as increases return.

As the hedge ratio increases from 2.4%, the MCCVaR for both asset-specific hedging assets increases as the risk diversifying effect of FX exposure becomes stronger. However, the MCCVaR for the global equity hedging asset increases faster than that for the global bond hedging asset, making it less attractive to hedge at 50% and 100% hedge ratios. This is consistent with the optimal hedging ratios under the asset-specific hedging constraint shown in Exhibit 4. Initially, only the hedge ratio for global equities increases, as it offers a better risk/return trade-off than hedging global bonds. As hedging global equities becomes less attractive than hedging global bonds, it is gradually replaced by the latter, until the latter reaches the upper bound. After that, it takes more hedging of global equities to increase return further, until it reaches its own upper bound.

---

Exhibit 4: Optimal asset-specific hedge ratios along the efficient frontier for the AUD case

<table>
<thead>
<tr>
<th>Hedge ratio</th>
<th>CVaR for optimized portfolio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>0.4</td>
<td>13</td>
</tr>
<tr>
<td>0.6</td>
<td>14</td>
</tr>
<tr>
<td>0.8</td>
<td>15</td>
</tr>
<tr>
<td>1.0</td>
<td>16</td>
</tr>
</tbody>
</table>

Hedge ratio

- GL equity hedging
- GL bond hedging

Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.
Source: PIMCO as of January 2016

(3) Currency-specific hedge ratios

This is the least restrictive optimization among the three.

Exhibit 5: Marginal risk and return contributions of currency-specific hedging for 0% and 100% hedged AUD 60/40 portfolio

<table>
<thead>
<tr>
<th>Marginal contribution to return (%)</th>
<th>EUR hedging</th>
<th>USD hedging</th>
<th>JPY hedging</th>
<th>GBP hedging</th>
<th>CHF hedging</th>
<th>CAD hedging</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.
Source: PIMCO as of January 2016
Exhibit 5 shows the marginal risk and return contributions of currency-specific hedging assets for portfolios with uniform hedge ratios of 0% and 100%. The marginal contributions to CVaR for all the currencies strictly increase as the hedging ratio increases. This is because FX exposure is more likely to be a risk diversifier than a risk contributor. This effect is most pronounced for USD, which is expected given the currency’s dominant weight in the unhedged portfolio (Exhibit 20 in the appendix).

For the 0% hedged portfolio, JPY, CHF and EUR serve as risk diversifiers, and hedging them increases the portfolio risk. On the other hand, USD, GBP and CAD are risk contributors, and hedging them reduces the portfolio risk. This explains why the unhedged portfolio sits well below the efficient frontier with currency-specific hedging constraints (Exhibit 1), because hedging these three currencies starting from the unhedged portfolio may boost return while reducing risk.

As the hedge ratio increases from 0% to 100%, all currencies are risk diversifiers, and hedging any FX marginally increases portfolio risk. The hedging attractiveness based on the marginal risk/return trade-off is similar across different hedge ratios for most of the currencies, except for USD, which quickly becomes the least attractive to hedge as the hedge ratio increases.

Given the diverse risk/return trade-offs offered by different currencies, it is reasonable to expect a relatively big efficiency loss to impose uniform or asset-specific hedging constraints because they prevent the optimizer from selectively hedging individual currencies based on their attractiveness.

Exhibit 6 shows the optimal currency-specific hedge ratios along the corresponding efficient frontier. CAD and GBP remain 100% hedged, as hedging them offers the best risk/return trade-offs at different levels of hedging, as indicated by Exhibit 5. The attractiveness of hedging USD decreases dramatically as the overall hedge ratio increases, due to a stronger diversification effect of USD versus AUD. As a result, USD is the last FX to be fully hedged. Hedging JPY is also relatively unattractive, and JPY is the second last to be fully hedged.

In this particular example, the incremental efficiency gain of relaxing the currency hedging constraint from asset-specific hedge ratios to currency-specific is large compared with that from a uniform hedge ratio to asset-specific ones.

### 2.2 JPY Base Case

The efficient frontiers under the three types of portfolio-level currency hedging constraints are shown in Exhibit 7. Similar to the AUD case, there is a relatively large efficiency gain from replacing the asset-specific hedge ratio constraints with currency-specific ones, compared with that of replacing uniform hedge ratio constraints with asset-specific ones.
Unlike the AUD case, the unhedged global portfolio is now at the far right end of the efficient frontier under the uniform hedge ratio constraint. This indicates the overall currency exposures in the unhedged portfolio are risk contributors. This is consistent with the notion that JPY is a risk-off currency that tends to appreciate during equity sell-offs. Next, we characterize the individual frontiers:

(1) Uniform hedge ratio

Exhibit 8 shows that the marginal contribution to CVaR of the uniform hedge ratio is negative for portfolios that are from 0% to 100% hedged.

(2) Asset-specific hedge ratios

Exhibit 9 shows the marginal contributions to portfolio return and CVaR of different hedging assets for three levels of uniform hedging (0%, 50% and 100%).
The three hedging assets are in the third quadrant, with negative marginal contributions to both return and risk of the portfolio (please see Exhibit 22 for the expected returns on the hedging assets for the JPY case). Exhibit 9 shows that hedging global bonds is more attractive than hedging global equity for all three hedge ratios. This explains the optimal hedge ratios along the efficient frontier where global equity hedging is reduced before global bond hedging (Exhibit 10).
CHF is the only hedging asset in the second quadrant. It has positive return contribution and negative risk contribution. The unique risk/return trade-off offered by CHF hedging at the 0% hedge ratio indicates there is a way to increase return while reducing risk at this point if each FX is allowed to have its own hedge ratio. All the currency hedging assets other than CHF hedging are in the third quadrant, with negative marginal contributions to both return and risk. The attractiveness of the hedged assets in the third quadrant can be ranked roughly in the following order: EUR>GBP>CAD>AUD>USD.

Exhibit 12 shows the currency-specific hedge ratios along the efficient frontier. Since USD hedging is the least attractive, the USD hedge ratio is the first to be reduced as the optimizer moves away from the minimum-CVaR portfolio, which is 100% hedged. After the USD hedge ratio reaches its zero lower bound, the hedge ratios for currencies other than CHF are sequentially reduced based on the attractiveness of the risk/return trade-offs they offer. CHF remains fully hedged at the upper bound along the frontier because additional hedging of this currency always increases return while reducing risk, as indicated by Exhibit 11.

2.3 USD Base Case

Exhibit 13 plots the efficient frontier for the hedging optimization with currency-specific hedging constraints only for USD investors. The optimizations with a uniform hedge ratio or asset-specific hedge ratio constraints do not result in efficient frontiers in the conventional sense. This is because under the model assumptions, hedging more currency exposure proportionally by portfolio or by assets always results in higher return and lower risk for the portfolio. As a result, only one optimal portfolio is considered efficient for either the uniform hedge ratio or the asset-specific hedge ratios: the minimum-CVaR portfolio, which also has the highest return among all optimized portfolios.

Exhibit 13: Efficient frontier for USD investors

<table>
<thead>
<tr>
<th>Estimated return (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
</tr>
<tr>
<td>3.5</td>
</tr>
<tr>
<td>3.0</td>
</tr>
<tr>
<td>2.5</td>
</tr>
</tbody>
</table>

We characterize the individual optimization setups as:

(1) Uniform hedge ratio

Exhibit 14 shows that the marginal contribution to CVaR of the uniform hedge ratio is negative for portfolios from 0% to 100% hedged.
Exhibit 14: MCCVaR of uniform hedging ratio for the USD global 60/40 portfolio

MCCVaR
-0.020
-0.025
-0.030
-0.035
-0.040
-0.045
-0.050

Uniform hedge ratio

Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.
Source: PIMCO as of January 2016

Exhibit 24 in the appendix shows the expected return for the portfolio hedging asset is 0.7%, so hedging reduces risk while increasing the expected return of the portfolio.

(2) Asset-specific hedge ratios

Exhibit 15 shows the marginal contributions to portfolio return and CVaR of different hedging assets at three levels of uniform hedging (0%, 50% and 100%).

The three hedging assets are all in the second quadrant, with negative MCCVaR and positive MCR. The efficient frontiers for optimizations under uniform or asset-specific hedge ratio constraints will be reduced to the 100% hedged portfolio.

(3) Currency-specific hedge ratios

Exhibit 16 shows the marginal risk and return contributions of currency-specific hedging assets for the 0% and 100% hedged portfolios. For all portfolios, JPY is the only risk diversifier among all the FXs considered. This is consistent with USD being a safe haven currency, which tends to appreciate against most currencies except JPY during down markets. Hedging JPY increases both return and risk of the portfolios.

All the other currencies are risk contributors. Among them, AUD is the only hedging asset with negative marginal contribution to return by assumption. The other currency hedging assets are in the second quadrant and are therefore most desirable, because hedging them may both increase return and reduce risk.

Exhibit 16: Marginal risk and return contributions of currency-specific hedging for 0% and 100% hedged USD 60/40 portfolio

Marginal contribution to return (%)
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30

Marginal contribution to CVaR (%)
0.00
0.5
1.0
1.5

Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.
Source: PIMCO as of January 2016

The risk/return trade-offs offered by hedging JPY or AUD are different from those offered by hedging the other currencies, which are all in the second quadrant (MCR>0, MCCVaR<0).
This allows the existence of an efficient frontier under the currency-specific hedge ratio constraint. Starting from the 100% hedged portfolio, there is no way to increase return without increasing risk. This means the 100% hedged portfolio is on the efficient frontier. From this point, we can either reduce the JPY hedge ratio, which will decrease both risk and return, or reduce the AUD hedge ratio, which will increase both risk and return. The minimum-CVaR portfolio is therefore a portfolio with JPY 0% hedged and all the other FXs 100% hedged. When the optimizer moves away from the minimum-CVaR portfolio toward a higher return target, it can either increase the hedge ratio of JPY from 0% or reduce the hedge ratio of AUD from 100%. Exhibit 17 shows the currency-specific hedge ratios on the efficient frontier, which are consistent with this intuition.

Exhibit 17: Optimal currency-specific hedge ratios along the efficient frontier

In our examples, if a currency hedging asset is in the second quadrant of the marginal risk and return contributions space (MCR>0, MCCVaR<0), the corresponding currency adds “risk without reward.” For this to happen collectively across most FX exposures, the base currency has to be a safe haven currency so that most foreign currencies serve as risk contributors. In addition, the expected returns of these foreign currencies have to be negative or close to zero. This is possible if the base currency has higher current or future interest rates relative to the other currencies or is expected to appreciate during the investment horizon. In our examples, JPY satisfies the first condition but not the second. AUD satisfies the second condition but not the first. USD, under our assumptions, satisfies both conditions. As a result, the efficient frontiers under uniform and asset-specific hedging constraints for the USD base case do not exist.

RISK MINIMIZING PORTFOLIOS AND RISK SPECTRUM OF CURRENCIES

As with the mean-variance optimization, we expect mean-CVaR optimization to be sensitive to estimation errors in the inputs, especially in means. To minimize the impact of return assumptions on the optimized allocation, we show in Exhibit 18 the hedging profiles of the minimum-CVaR portfolios, which are independent of the expected return assumptions \( E(R^p_i) = 0 \) for all \( i \). Here the mean-CVaR optimization will have only one optimal solution: the minimum-CVaR portfolio.

Exhibit 18: Hedge ratios in the minimum-CVaR portfolios

The interesting result for the USD base case illustrates how FX exposures can collectively add meaningful risk to the global 60/40 portfolio without providing positive risk premia to compensate the investors. Although our results are based on hypothetical expected return assumptions, Boudoukh, Katz, Richardson and Thapar (2015) had similar findings using historical data for unhedged and hedged global equity portfolios for U.S.-based investors.
Intuitively, to minimize risk, one should only hedge the currencies that tend to depreciate relative to the base currency when the portfolio experiences large drawdowns. We can rank the major DM currencies according to their relative “risk-on-off-ness” based on the signs of downside correlations between each of the individual bilateral exchange rates and the MSCI World Local Index return since 1997:

The hedging profiles for the minimum-CVaR portfolios are broadly in line with this risk spectrum, because equity risk dominates the 60/40 portfolios. For JPY investors, all other currencies are relatively risk-off and will contribute to portfolio risk if left unhedged. For USD investors, all currencies except for JPY are relatively risk-on, and hedging them all would minimize the portfolio risk. For AUD investors, most currencies are relatively risk-off, and hedging them would likely increase risk.4

**CONCLUSIONS**

Many institutional investors with global multi-asset portfolios impose uniform hedge ratios at the portfolio or asset level. We introduce an analytical framework based on marginal risk/return trade-offs to quantify and explain the efficiency loss due to the common practices. We use the mean-CVaR optimization framework to capture tail risk with non-normal returns. To emphasize the importance of base currency for the currency hedging optimization, we perform sample optimizations for AUD-, JPY- and USD-based investors.

**References**


### Appendix 1 Model Assumptions

#### Exhibit 19: Global 60/40 portfolio and capital market assumptions: AUD base

<table>
<thead>
<tr>
<th>Asset</th>
<th>Proxy</th>
<th>60/40 Portfolio</th>
<th>10-Year CMAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global equities</td>
<td>MSCI World Ex-Australia</td>
<td>30%</td>
<td>4.73%</td>
</tr>
<tr>
<td>Global bonds</td>
<td>Barclays Global Aggregate</td>
<td>20%</td>
<td>1.50%</td>
</tr>
<tr>
<td>Domestic equities</td>
<td>S&amp;P/ASX 300</td>
<td>30%</td>
<td>5.37%</td>
</tr>
<tr>
<td>Domestic bonds</td>
<td>Bloomberg AusBond Index</td>
<td>20%</td>
<td>2.94%</td>
</tr>
</tbody>
</table>

*Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.*

#### Exhibit 20: Hedging assets construction: AUD base

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>EUR</th>
<th>JPY</th>
<th>GBP</th>
<th>CAD</th>
<th>CHF</th>
<th>Expected return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency hedging ret (Ri)</td>
<td>0.69%</td>
<td>2.51%</td>
<td>2.63%</td>
<td>1.47%</td>
<td>1.26%</td>
<td>3.94%</td>
<td></td>
</tr>
<tr>
<td>FX exposures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio</td>
<td>26.82%</td>
<td>8.61%</td>
<td>5.78%</td>
<td>3.78%</td>
<td>1.68%</td>
<td>1.28%</td>
<td></td>
</tr>
<tr>
<td>Global equities</td>
<td>17.72%</td>
<td>3.59%</td>
<td>2.73%</td>
<td>2.45%</td>
<td>1.12%</td>
<td>1.13%</td>
<td></td>
</tr>
<tr>
<td>Global bonds</td>
<td>8.98%</td>
<td>4.86%</td>
<td>3.05%</td>
<td>1.25%</td>
<td>0.54%</td>
<td>0.16%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>USD</td>
<td>EUR</td>
<td>JPY</td>
<td>GBP</td>
<td>CAD</td>
<td>CHF</td>
<td>Expected return</td>
</tr>
<tr>
<td></td>
<td>-26.82%</td>
<td>-8.61%</td>
<td>-5.78%</td>
<td>-3.78%</td>
<td>-1.68%</td>
<td>-1.28%</td>
<td>-0.57%</td>
</tr>
</tbody>
</table>

*Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.*

#### Exhibit 21: Global 60/40 portfolio and capital market assumptions: JPY base

<table>
<thead>
<tr>
<th>Asset</th>
<th>Proxy</th>
<th>60/40 Portfolio</th>
<th>10-Year CMAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global equities</td>
<td>MSCI World Ex-Japan</td>
<td>30%</td>
<td>5.20%</td>
</tr>
<tr>
<td>Global bonds</td>
<td>Barclays Global Agg ExJpn</td>
<td>20%</td>
<td>2.55%</td>
</tr>
<tr>
<td>Domestic equities</td>
<td>MSCI Japan Index</td>
<td>30%</td>
<td>4.32%</td>
</tr>
<tr>
<td>Domestic bonds</td>
<td>JPM Japan Govt Bond Index</td>
<td>20%</td>
<td>0.52%</td>
</tr>
</tbody>
</table>

*Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.*

#### Exhibit 22: Hedging assets construction: JPY base

<table>
<thead>
<tr>
<th></th>
<th>USD</th>
<th>EUR</th>
<th>GBP</th>
<th>CAD</th>
<th>AUD</th>
<th>CHF</th>
<th>Expected return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Currency hedging ret (Ri)</td>
<td>-1.94%</td>
<td>-0.12%</td>
<td>-1.16%</td>
<td>-1.37%</td>
<td>-2.63%</td>
<td>1.31%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>USD</td>
<td>EUR</td>
<td>GBP</td>
<td>CAD</td>
<td>AUD</td>
<td>CHF</td>
<td>Expected return</td>
</tr>
<tr>
<td></td>
<td>-29.50%</td>
<td>-9.58%</td>
<td>-4.08%</td>
<td>-1.84%</td>
<td>-1.21%</td>
<td>-0.03%</td>
<td></td>
</tr>
</tbody>
</table>

*Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.*
Hypothetical example for illustrative purposes only. Refer to Appendix for additional CVaR, hypothetical example and return assumptions information.

| Exhibit 23: Global 60/40 portfolio and capital market assumptions: USD base |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| Asset                      | Proxy                        | 60/40 Portfolio             | 10-Year CMAs                |
| Global equities            | MSCI World ex-U.S. Index     | 30%                         | 3.75%                       |
| Global bonds               | Barclays Global Agg ex-USD   | 20%                         | 0.31%                       |
| Domestic equities          | S&P 500 Index                | 30%                         | 4.50%                       |
| Domestic bonds             | Barclays U.S. Aggregate Index | 20%                         | 2.49%                       |

The return measured in the base currency on a currency hedged portfolio is

\[ R_P = R_U + R_H \]

where \( R_U \) is the return on the unhedged portfolio and \( R_H \) is the return on the currency hedging overlay, which consists of short positions in forward contracts on various foreign currencies. Thus,

\[ R_U = \sum_{i=0}^{N} \left( w_i(B) R_i(B) + w_i(E) R_i(E) \right) \]

where \( R_i(B) \) and \( R_i(E) \) denote the total returns in base currency on the bonds and equities, respectively, in currency \( i (i = 0, ..., N) \) with \( i = 0 \) denoting the base currency. Moreover, \( w_i(B) \) and \( w_i(E) \) denote the proportions of bonds and equities in currency \( i \) in the investor’s portfolio. Thus, \( \sum_{i=0}^{N} \left( w_i(B) + w_i(E) \right) = 1 \).

Since base currency returns can be decomposed into local currency returns and FX spot returns, we also have

\[ R_U = \sum_{i=0}^{N} \left( w_i(B) R_i^f(B) + w_i(E) R_i^f(E) \right) \]

\[ + \sum_{i=1}^{N} \left( (w_i(B) + w_i(E)) R_i^S \right) \]

where \( R_i^f(B) \) and \( R_i^f(E) \) are local currency returns on bonds and equities in currency \( i (i = 0, ..., N) \) and \( R_i^S \) is the spot return for currency \( i (i = 1, ..., N) \). Thus, the exposure of the investor to currency \( i \) is \( w_i = w_i(B) + w_i(E) \).

The return on currency hedging overlay \( R_i^H \) is given by

\[ R_i^H = \sum_{i=1}^{N} \left( h_i w_i R_i^H \right) \]

where \( R_i^H \) is the payoff on a unit notional short position in the forward contract on currency \( i \), i.e.

\[ R_i^H = r_0 - r_i - R_i^S \]
where $r_a$ and $r_i$ ($i = 1, ..., N$) are short-term riskless rates in the base currency and foreign currency $i$. The FX hedging return is determined by both the spot returns and the forward premium, which depends on the risk-free rate differential between the FX and the base currency.

Moreover, $h_i$ is the proportion of currency exposure $w_i$ the investor actually hedges. If $h_i = 1$ for all currencies, then all currency exposure is hedged. In the optimization problems considered in this paper, we impose the constraint that $0 \leq h_i \leq 1$. No over hedging or doubling up on currency risk is permitted.

We consider three cases in increasing order of complexity:

1. Uniform hedging: $h_i = h_A$ for all $i$ where the universal hedging proportion $h_A$ is chosen optimally.

2. Asset-specific hedging: $h_i = h_B w(B) + h_E w(E)$ where a fraction $h_B$ of all overseas bond holding is hedged and a fraction $h_E$ of all overseas equity holdings is hedged. The hedging proportions $h_B$ and $h_E$ are optimally chosen.

3. Currency-specific hedging: $h_i$'s are unrestricted other than the constraint that $0 \leq h_i \leq 1$.

Mathematically, we construct mean-CVaR efficient frontiers by solving the following optimization problem at different levels of target portfolio return $c$:

$$\min_{h=(h_i)_{i=1}^N} \text{CVaR}(h)$$

subject to

$$ER_B + ER_E(h) = c,$$

$$0 \leq h_i \leq 1, i = 1, 2, ..., N.$$ 

As discussed previously, the control variable $h = (h_i)_{i=1}^N$ may be subject to additional constraints under certain setups. Given the uniform hedging constraint, $h_i = h_A$ for all $i$. Under the asset-specific hedging constraint, $h_i = h_B w(B) + h_E w(E)$ where a fraction $h_B$ of all overseas bond holding is hedged and a fraction $h_E$ of all overseas equity holdings is hedged.

PIMCO Capital Market Assumptions are as of May 2015. Currency exposure data is as of 31 March 2016.
Campbell et al. (2010) find that a risk minimizing global equity investor should short the Australian dollar, Canadian dollar, Japanese yen and British pound and long the U.S. dollar, euro and Swiss franc, but for a global bond investor, the risk minimizing strategy is a fully hedged position; Schmittmann (2010) finds that for German, Japanese, UK and U.S. bond investors, full currency hedging is the risk minimizing strategy, but for equities the risk minimizing strategy is unhedged exposure to developed market (DM)/euro and zero currency exposure to the yen and U.S. dollar.

The standard hedging practice for Australian superannuation funds is an average 34% hedging for global equities and 100% hedging for global bonds (Chant, Mohankumar and Warren (2014)); Chilean private pension funds (a.k.a. administradoras de fondos de pensiones, or AFPs) have asset-specific regulatory guidelines on currency hedging (OECD, 2015).

Chopra and Ziemba (1993) found that errors in means were about 10 times as important as errors in variances for a risk tolerance typical for institutional investors.

CAD and GBP are the only exceptions. They are slightly more risk-off than AUD based on the downside correlations between their exchange rates against AUD and world equity. As seen in Exhibit 5, they can be risk contributors when the 60/40 portfolio is unhedged.
Past performance is not a guarantee or a reliable indicator of future results.

Conditional Value at Risk (CVAR) estimates the risk of loss of an investment or portfolio over a given time period under normal market conditions in terms of an average of loss after a specific percentile threshold of loss (i.e., for a given threshold of X%, under the specific modeling assumptions used, the portfolio will incur an average loss in excess of the CVAR X percent of the time). Different CVAR model methodologies can help understand what future returns or loss profiles might be. However, the effectiveness of a CVAR calculation is in fact constrained by its limited assumptions (for example, assumptions may involve, among other things, probability distributions, historical return modeling, factor selection, risk factor correlation, simulation methodologies). It is important that investors understand the nature of these limitations when relying upon CVAR analyses.

No representation is being made that any account, product, or strategy will or is likely to achieve profits, losses, or results similar to those shown. Hypothetical or simulated performance results have several inherent limitations. Unlike an actual performance record, simulated results do not represent actual performance and are generally prepared with the benefit of hindsight. There are frequently sharp differences between simulated performance results and the actual results subsequently achieved by any particular account, product or strategy. In addition, since trades have not actually been executed, simulated results cannot account for the impact of certain market risks such as lack of liquidity. There are numerous other factors related to the markets in general or the implementation of any specific investment strategy, which cannot be fully accounted for in the preparation of simulated results and all of which can adversely affect actual results.

Return assumptions are for illustrative purposes only and are not a prediction or a projection of return. Return assumption is an estimate of what investments may earn on average over a ten year period. Actual returns may be higher or lower than those shown and may vary substantially over shorter time periods.

Investing in the bond market is subject to risks, including market, interest rate, issuer, credit, inflation risk, and liquidity risk. The value of most bonds and bond strategies is impacted by changes in interest rates. Bonds and bond strategies with longer durations tend to be more sensitive and volatile than those with shorter durations; effect of changes in interest rates on the value of shorter duration bonds is expected to be less than for longer duration securities. Interest rate changes may cause principal of bond investments to decline in value. Bond investments may be worth more or less than the original cost when redeemed. Investing in foreign denominated and/or domiciled securities may involve heightened risk due to currency fluctuations, and economic and political risks, which may be enhanced in emerging markets. Mortgage and asset-backed securities may be sensitive to changes in interest rates, subject to early repayment risk, and their value may fluctuate in response to the market's perception of issuer creditworthiness; generally supported by some form of government or private guarantee there is no assurance that private guarantors will meet their obligations. High-yield, lower-rated securities involve greater risk than higher-rated securities; portfolios that invest in these may be subject to greater levels of credit and liquidity risk than portfolios that do not. Currency rates may fluctuate significantly over short periods of time and may reduce the returns of a portfolio. Equities may decline in value due to both real and perceived general market, economic, and industry conditions. Tail risk hedging may involve entering into financial derivatives that are expected to increase in value during the occurrence of tail events. Investing in a tail event instrument could lose all or a portion of its value even in a period of severe market stress. A tail event is unpredictable; therefore, investments in instruments tied to the occurrence of a tail event are speculative. Derivatives may involve certain costs and risks such as liquidity, interest rate, market, credit, management and the risk that a position could not be closed when most advantageous. Investing in derivatives could lose more than the amount invested.

It is not possible to invest directly in an unmanaged index.

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